# Sec. 9.2 Sum and Difference Formulas for Sine and Cosine

***Sum and Difference Formulas***

$\sin(\theta + \varphi) = \sin\theta\cos\varphi + \cos\theta\sin\varphi$

$\sin(\theta - \varphi) = \sin\theta\cos\varphi - \cos\theta\sin\varphi$

$\cos(\theta + \varphi) = \cos\theta\cos\varphi - \sin\theta\sin\varphi$

$\cos(\theta - \varphi) = \cos\theta\cos\varphi + \sin\theta\sin\varphi$

**Ex:** Find an exact value for $\cos 15^\circ$.

$\cos(60^\circ - 45^\circ) = \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

**Ex:** Find an identity for $\sin 3\theta$ in terms of $\sin\theta$ and $\cos\theta$.

$\sin(2\theta + \theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= 2\sin\theta\cos\theta\cos\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$

$= 2\cos^2\theta\sin\theta + \cos^2\theta\sin\theta - \sin^3\theta$

$= \boxed{3\cos^2\theta\sin\theta - \sin^3\theta}$

– OR –

$2\sin\theta\cos\theta\cos\theta + (2\cos^2\theta - 1)\sin\theta$

$2\cos^2\theta\sin\theta + 2\cos^2\theta\sin\theta - \sin\theta$

$\boxed{4\cos^2\theta\sin\theta - \sin\theta}$

**Ex:** Using the sum and difference formulas, check the following results.

a. $\cos(t - \pi/2) = \sin t$

$\cos t\cos\frac{\pi}{2} + \sin t\sin\frac{\pi}{2}$

$\cos t\cdot 0 + \sin t(1)$

$\sin t$

b. $\sin(t + \pi/2) = \cos t$

$\sin t\cos\frac{\pi}{2} + \cos t\sin\frac{\pi}{2}$

$\sin t(0) + \cos t\cdot 1$

$\cos t$

Provided their periods are equal, the sum of a sine function and a cosine function can be written as a single sinusoidal function. We have

$$a_1 \sin(Bt) + a_2 \cos(Bt) = A \sin(Bt + \varphi)$$

where $A = \sqrt{a_1^2 + a_2^2}$ and $\varphi = \tan^{-1} = \dfrac{a_2}{a_1}$

The angle φ is determined by the equations

$$\cos \varphi = a_1 / A \quad \text{and} \quad \sin \varphi = a_2 / A.$$

**Ex:** Check algebraically that $\sin t + \cos t = \sqrt{2} \sin(t + \pi/4)$.

$a_1 = 1 \quad a_2 = 1 \quad B = 1$

$A = \sqrt{1^2 + 1^2}$

$A = \sqrt{2}$

$\phi = \tan^{-1}\left(\frac{1}{1}\right)$

$\phi = \tan^{-1}(1)$

$\phi = \frac{\pi}{4}$

$A \sin(Bt + \phi)$

$1 \sin\left(t + \frac{\pi}{4}\right)$

$\sin\left(t + \frac{\pi}{4}\right)$

$\cos \phi = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin \phi = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

QI*

**Ex:** If $g(t) = 2\sin(3t) + 5\cos(3t)$, write $g(t)$ as a sinusoidal function.

$a_1 = 2 \quad a_2 = 5 \quad B = 3$

$A = \sqrt{2^2 + 5^2}$

$A = \sqrt{29}$

$\phi = \tan^{-1}\left(\frac{5}{2}\right)$

$\phi = 1.190$ (QI)

$g(t) = \sqrt{29} \sin(3t + 1.190)$

$\cos \phi = \frac{2}{\sqrt{29}}$

$\sin \phi = \frac{5}{\sqrt{29}}$

QI*

*Sum and Difference of Sine and Cosine*

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

**Ex:** Write $\cos(30t) + \cos(28t)$ as the product of two cosine functions.

$$2\cos\left(\frac{30t+28t}{2}\right)\cos\left(\frac{30t-28t}{2}\right)$$

$$2\cos(29t)\cos t$$

HW: pg 383-385 #1,3,5(75 only),6-8,10,18

REVIEW FOR QUIZ